hep-th/0804.1994

Supersymmetric and nonsupersymmetric perturbations to KT

Shesansu Sekhar Pal

Barchana, Jajpur 754081, Orissa, India
shesansu AT gmail.com

Abstract

We studied the supersymmetric and non-supersymmetric perturbations to cascading gauge theory. In particular we use KT background and the back reaction of the generic linearized perturbation make the dilaton to run and the $T^{(1,1)}$ gets squashed which in turn make the supersymmetry to be broken. But if we make a special linearized perturbation in such way that the $T^{(1,1)}$ is not squashed then the corresponding perturbation preserve supersymmetry.

1 Introduction

It has been a practice to generate as much gravity solution as possible. As these solutions are of prime importance because of the celebrated gauge gravity duality [1], [2], [3]. Even though this duality is for the $AdS_5 \times S^5$ and the $\mathcal{N} = 4$ in 3+1 dimension, but still there is a hope that we can apply this duality for different cases in the sense of non-conformal, supersymmetric and non-supersymmetric solutions.

In this context there arises an interesting gravity solution generated by Klebanov and collaborators, (Nekrasov, Tseytlin and Strassler) [4],[5],[7], who considered the supersymmetric intersections of N D3 branes and M D5 branes wrapped on the 2-cycle of a Calabi-Yau manifold that is the conifold [8]. For this configuration they successfully generated an interesting and important gravity solutions which preserves $\mathcal{N} = 1$ supersymmetry and are non-conformal in nature. The most interesting solution that is [7] shows confinement but in the large r limit it goes over to the singular [5] solution. The corresponding dual field theory is the $\mathcal{N} = 1$ supersymmetric $SU(N + M) \times SU(N)$ gauge theory with two bi-fundamental and two anti-bifundamental chiral superfields and with a non-trivial superpotential. A T-dual version of this is studied by [13],[14], in which the authors took the intersection of two stacks of separated NS 5 branes extended along (12345) and (12389) and a stack of D4 branes along (1236). The coordinate x^6 is taken as compact and the two gauge couplings are determined by the position of the NS 5 brane, in particular the couplings are equal when they are placed at the diametrically opposite points.

These solutions has the interesting feature like the cascading behavior [9], which means the rank of the gauge group falls and become $SU(N - M) \times SU(M)$ gauge theory after going through one Seiberg duality and this process continues until IR where it reaches a confining gauge theory described by SU(M) gauge theory. The duality is properly constructed from the field theory point of view in [7], [10] and its being emphasized in [10] that this duality is not a property at IR but its an exact duality.

The gauge theory has got two couplings $g_1, \;\; g_2$ apart from the coupling λ that comes from the superpotential. These two gauge couplings are related to the dilaton and the integrated 2-from flux that comes from NS-NS sector over the 2-cycle of $T^{(1,1)}$ by gauge/gravity duality, whose precise form is given in [7], [10]. Topologically $T^{(1,1)}$ is $S^2 \times S^3$ with symmetry $SU(2) \times SU(2) \times U(1)$,

and is a coset space $\frac{SU(2)\times SU(2)}{U(1)}$.

From the gravity point of view the solutions [5], [7] has got a non-trivial 3-from complex field strength, G_3, which has the structure (2,1), according to its Hodge classification [7], [11] and obey the ISD condition. This particular structure of the flux in turn make the dilaton to be a constant and the brane configuration supersymmetric. But its not necessarily true that by making the dilaton to run we have to break supersymmetry. We shall see both the supersymmetric and non-supersymmetric (linearized) solution for which the dilaton is not constant.

The metric of the KT solution preserves a U(1) symmetry, which is associated to the shift of an angle connected to the fiber of $T^{(1,1)}$, but is broken to Z_{2M} by the 2-form potential coming from the RR sector, even though its field strength does preserve this symmetry. This in the dual field theory is interpreted as the break down of the R-symmetry [12] and its a spontaneous breaking. From this it follows that in the absence of D5 branes wrapped on the 2-cycle this U(1) symmetry is an exact symmetry. In the linearized perturbed solution, we shall see explicitly that the whole things goes over and the R-symmetry is broken irrespective of whether we break supersymmetry or not, as we do not change the 2-form RR potential, C_2.

Summarizing, in this paper, we shall see the following points.

(1) By doing a linearized perturbation to the metric and fluxes with two independent parameters $\mathcal{S}$ and ϕ, [16], we considered the back reaction of this perturbation to the original KT solution and obtained the solution in Einstein frame as

$$\begin{aligned}
ds^2 &= h^{-\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + h^{\frac{1}{2}}[dr^2 + r^2(1+\frac{16}{81}\frac{h_2\mathcal{S}}{r^4})\frac{g_5^2}{9} + \frac{r^2}{6}(g_1^2+g_2^2+g_3^2+g_4^2)] \\
F_3 &= \frac{M\alpha'}{4}g_5\wedge(g_1\wedge g_2 + g_3\wedge g_4), \quad C_0 = 0 \\
B_2 &= \frac{g_sM\alpha'}{2}f(r)(g_1\wedge g_2+g_3\wedge g_4),\ H_3 = \frac{g_sM\alpha'}{2}f'(r)dr\wedge(g_1\wedge g_2+g_3\wedge g_4) \\
\widetilde{F}_5 &= (1+\star_{10})\mathcal{F}_5, \quad \mathcal{F}_5 = \frac{g_sM^2\alpha'^2}{4}\ell(r)g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5 \\
h(r) &= \frac{27\pi g_s\alpha'^2}{4r^4}[\frac{3g_sM^2}{2\pi}Log\ \frac{r}{r_0} + \frac{3g_sM^2}{8\pi}] + \frac{(g_SM\alpha')^2}{r^8}[(h_1+h_2Log\ r)\mathcal{S} - h_3\phi] \\
\Phi(r) &= Log\ g_s + \frac{1}{r^4}\Big((-\frac{64}{81}h_1+\frac{52}{81}h_2)\mathcal{S} - \frac{16}{27}h_2\mathcal{S}Log\ r + \frac{64}{81}h_3\phi\Big)
\end{aligned}$$

$$f(r) \;=\; k(r) = \ell(r) = \frac{3}{2}Log\ r + \frac{1}{r^4}\Big((\frac{8}{27}h_1 - \frac{h_2}{27})\mathcal{S} + \frac{4}{9}\mathcal{S}h_2 Log\ r - \frac{8}{27}h_3\phi\Big), \tag{1}$$

where $h_1,\ h_2$ and h_3 are real and independent parameters. For a very specific choice like: $h_1 = \frac{1053}{256},\ \ h_2 = \frac{81}{16}$, and $h_3 = \frac{81}{64}$, we get the solution [16].

(2)From this solution it just follows that we have not changed the RR 3-form, F_3, flux and hence the quantization condition associated to it remains intact. But the H_3 is now different so also the 5-form flux. The corresponding quantization condition has the leading term which goes logarithmically but the subleading term goes as inverse power law for $h_2 = 0$ and in general

$$\oint_{T^{(1,1)}} \widetilde{F_5} \sim g_s M^2 \alpha'^2 \Big[\frac{3}{2}Log\ r + \frac{1}{r^4}\Big((\frac{8}{27}h_1 - \frac{h_2}{27})\mathcal{S} + \frac{4}{9}\mathcal{S}h_2 Log\ r - \frac{8}{27}h_3\phi\Big)\Big] \tag{2}$$

(3) The complex 3-from flux made out of NS-NS, RR 3-from fluxes and the dilaton contains both the imaginary self dual and the imaginary anti self dual piece.

$$\begin{aligned} G_+ &= \frac{iM\alpha'}{r^9}\Big[2 - \frac{\mathcal{S}h_2}{r^4}\Big(\frac{28}{81} + \frac{16}{27}Log\ r\Big)\Big]\overline{z}_m dz_m \wedge \epsilon_{ijkl} z_i \overline{z}_j dz_k \wedge \overline{dz_l} \\ G_- &= -\frac{iM\alpha'}{r^9}\Big[\frac{\mathcal{S}h_2}{r^4}\Big(\frac{12}{81} + \frac{16}{27}Log\ r\Big)\Big] z_m \overline{dz}_m \wedge \epsilon_{ijkl} z_i \overline{z}_j dz_k \wedge \overline{dz}_l. \end{aligned} \tag{3}$$

It means generically the solution presented in eq(1) break supersymmetry. However for a very specific choice of parameter the solution preserves supersymmetry and it is $h_2 = 0$. Note that the squashing of $T^{(1,1)}$ is proportional to h_2.

(4) Upon looking at the space of solutions, it just follows that there is a supersymmetry preserving 2-plane described by (h_1, h_3) which sits at $h_2 = 0$. Away from this particular plane supersymmetry is broken and is broken dynamically.

(5)From the solution presented in eq(1), it just follows trivially that the dilaton runs irrespective of whether the solution is supersymmetric or not. But the way it runs depends very much on whether supersymmetry is preserved or not.

The organization of the paper is as follows. In section 2, we shall write down the equations of motion for IIB and the ansatz that we are going to use and in section 3, we shall give the details of the solutions and then conclude in section 4.

2 Ansatz and the equations

We would like to set up the notation for which we can apply the resulting equations for the deformed conifold. Let us introduce some 1-form objects following [6],[7]

$$\begin{aligned} g_1 &= \frac{e_1 - e_3}{\sqrt{2}}, \quad g_2 = \frac{e_2 - e_4}{\sqrt{2}} \\ g_3 &= \frac{e_1 + e_3}{\sqrt{2}}, \quad g_4 = \frac{e_2 + e_4}{\sqrt{2}}, \quad g_5 = e_5, \end{aligned} \tag{4}$$

where

$$\begin{aligned} &e_1 = -sin\theta_1 d\phi_1, \quad e_2 = d\theta_1, \quad e_3 = cos\psi\ sin\theta_2 d\phi_2 - sin\psi d\theta_2 \\ &e_4 = sin\psi\ sin\theta_2 d\phi_2 + cos\psi d\theta_2, e_5 = d\psi + cos\theta_1 d\phi_1 + cos\theta_2 d\phi_2. \end{aligned} \tag{5}$$

and construct an ansatz of 10-dim metric consistent with the symmetry of a deformed conifold

$$ds^2 = A^2(\tau)\eta_{\mu\nu}dx^\mu dx^\nu + B^2(\tau)d\tau^2 + C^2(\tau)g_5^2 + D^2(\tau)(g_3^2+g_4^2) + E^2(\tau)(g_1^2+g_2^2). \tag{6}$$

The 3-form fields, dilaton, axion and 5-form self-dual field is assumed to take the following form

$$\begin{aligned} F_3 &= \frac{M\alpha'}{2}[(1-F)g_5\wedge g_3\wedge g_4 + Fg_5\wedge g_1\wedge g_2 + F'd\tau\wedge(g_1\wedge g_3 + g_2\wedge g_4)] \\ B_2 &= \frac{g_sM\alpha'}{2}[f(\tau)g_1\wedge g_2 + k(\tau)g_3\wedge g_4], \quad C_0 = 0, \quad \Phi = \Phi(\tau) \\ H_3 &= \frac{g_sM\alpha'}{2}[d\tau\wedge(f'g_1\wedge g_2 + k'g_3\wedge g_4) + \frac{k-f}{2}g_5\wedge(g_1\wedge g_3 + g_2\wedge g_4)] \\ \tilde{F}_5 &= (1+\star_{10})\mathcal{F}_5, \quad \mathcal{F}_5 = \frac{g_sM^2\alpha'^2}{4}\ell g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5, \quad \ell = f(1-F) + kF \end{aligned} \tag{7}$$

Note that the H_3 we have taken is different from the ansatz considered in [24]. The difference is that our ansatz has a symmetry (ant-symmetric) under Z_2, which is to interchange the two S^2's i.e. (θ_1, ϕ_1) with (θ_2, ϕ_2). Its easy to check that the equation of motion remain unchanged under this.

For completeness we shall write down the ansatz to NS-NS 3-form field strength, written down in [24]

$$\begin{aligned} H_3^{(PT)} &= (h_2' - h_1')d\tau \wedge g_1 \wedge g_2 - (h_1' + h_2')d\tau \wedge g_3 \wedge g_4 - \\ &\quad h_2 g_5 \wedge (g_1 \wedge g_3 + g_2 \wedge g_4) + 2\chi' d\tau \wedge (g_2 \wedge g_3 - g_1 \wedge g_4), \end{aligned} \tag{8}$$

where we have taken τ as the radial coordinate.

This form of $H_3^{(PT)}$ has a piece which is symmetric and another piece anti-symmetric under Z_2, which means if we want to have the Z_2 symmetry in the equation of motion then we need to set χ' to zero. In any case we shall take the ansatz written above.

The dilaton equation in this case is

$$\nabla^2\Phi = -\frac{g_s e^{-\Phi}}{12}[H_{MNP}H^{MNP} - e^{2\Phi}F_{MNP}F^{MNP}]. \tag{9}$$

The various terms in this equation are

$$\begin{aligned} \nabla^2\Phi &= \frac{1}{A^4BCD^2E^2}\partial_\tau[\frac{A^4CD^2E^2\Phi'}{B}] \\ H_{MNP}H^{MNP} &\pm e^{2\Phi}F_{MNP}F^{MNP} = \frac{3}{2}M^2\alpha'^2[\frac{g_s^2 f'^2}{B^2E^4} + \frac{g_s^2 k'^2}{B^2D^4} + \frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2} \\ &\pm e^{2\Phi}\frac{(1-F)^2}{C^2D^4} \pm e^{2\Phi}\frac{F^2}{C^2E^4} \pm 2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}] \end{aligned} \tag{10}$$

So the dilaton equation of motion give

$$\begin{aligned} &\frac{1}{A^4BCD^2E^2}\partial_\tau[\frac{A^4CD^2E^2\Phi'}{B}] = -\frac{g_sM^2\alpha'^2}{8}e^{-\Phi}[\frac{g_s^2f'^2}{B^2E^4} + \frac{g_s^2k'^2}{B^2D^4} + \\ &\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2} - e^{2\Phi}\frac{(1-F)^2}{C^2D^4} - e^{2\Phi}\frac{F^2}{C^2E^4} - 2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}] \end{aligned} \tag{11}$$

The equation for F_3 is

$$d\star_{10}(e^{\Phi}F_3) = g_s\tilde{F}_5 \wedge H_3. \tag{12}$$

The expression for both the LHS and RHS are

$$
\begin{aligned}
d\star_{10}(e^{\Phi}F_3) &= \frac{M\alpha'}{2}[\frac{1-F}{2}e^{\Phi}\frac{A^4BE^2}{CD^2}-\frac{F}{2}e^{\Phi}\frac{A^4BD^2}{CE^2}+\partial_\tau(F'e^{\Phi}\frac{A^4C}{B})] \\
& d\tau\wedge g_5\wedge(g_1\wedge g_3+g_2\wedge g_4)\wedge dx^0\cdots\wedge dx^3 \\
g_s\tilde{F}_5\wedge H_3 &= \frac{(g_sM\alpha')^3}{8}\frac{A^4B}{CD^2E^2}\frac{\ell(k-f)}{2} \\
& d\tau\wedge g_5\wedge(g_1\wedge g_3+g_2\wedge g_4)\wedge dx^0\cdots\wedge dx^3
\end{aligned}
\tag{13}
$$

Hence the resulting equation of motion for F_3 is

$$
\frac{1-F}{2}e^{\Phi}\frac{A^4BE^2}{CD^2}-\frac{F}{2}e^{\Phi}\frac{A^4BD^2}{CE^2}+\partial_\tau(F'e^{\Phi}\frac{A^4C}{B})=\frac{g_s^3M^2\alpha'^2}{4}\frac{A^4B}{CD^2E^2}\frac{\ell(k-f)}{2}. \tag{14}
$$

The equation for H_3 is

$$
d\star_{10}(e^{-\Phi}H_3)=-g_s\tilde{F}_5\wedge F_3. \tag{15}
$$

The expression for both the LHS and RHS are

$$
\begin{aligned}
d\star_{10}(e^{-\Phi}H_3) &= -\frac{g_sM\alpha'}{2}[\partial_\tau(f'e^{-\Phi}\frac{A^4CD^2}{BE^2})+\frac{k-f}{2}e^{-\Phi}\frac{A^4B}{C}]g_5\wedge g_3\wedge g_4\wedge dx^0\cdots\wedge dx^3 \\
&- \frac{g_sM\alpha'}{2}[\partial_\tau(k'e^{-\Phi}\frac{A^4CE^2}{BD^2})-\frac{k-f}{2}e^{-\Phi}\frac{A^4B}{C}]g_5\wedge g_1\wedge g_2\wedge dx^0\cdots\wedge dx^3 \\
-g_s\tilde{F}_5\wedge F_3 &= \frac{g_s^2M^3\alpha'^3}{8}\frac{A^4B}{CD^2E^2}\ell dx^0\cdots\wedge dx^3\wedge d\tau\wedge[(1-F)g_5\wedge g_3\wedge g_4 \\
& +Fg_5\wedge g_1\wedge g_2]
\end{aligned}
\tag{16}
$$

Hence the resulting equation of motion for H_3 are

$$
\begin{aligned}
\frac{g_sM^2\alpha'^2}{2}\frac{A^4B}{CD^2E^2}\ell(1-F) &= 2\partial_\tau(f'e^{-\Phi}\frac{A^4CD^2}{BE^2})+(k-f)e^{-\Phi}\frac{A^4B}{C} \\
\frac{g_sM^2\alpha'^2}{2}\frac{A^4B}{CD^2E^2}F\ell &= 2\partial_\tau(k'e^{-\Phi}\frac{A^4CE^2}{BD^2})-(k-f)e^{-\Phi}\frac{A^4B}{C}.
\end{aligned}
\tag{17}
$$

The Bianchi identities associated to the form fields are identically satisfied.

Now the equation of motion for the metric results

$$
\begin{aligned}
R_{MN} &= \frac{1}{2}\partial_M\Phi\partial_N\Phi + \frac{g_s^2}{96}\tilde{F}_{MPQRS}\tilde{F}_N{}^{PQRS} + \frac{g_s e^{-\Phi}}{4}[H_{MPQ}H_N{}^{PQ} + e^{2\Phi}F_{MPQ}F_N{}^{PQ}] \\
&- \frac{g_s e^{-\Phi}}{48}g_{MN}[H_{PQR}H^{PQR} + e^{2\Phi}F_{PQR}F^{PQR}]. \qquad (18)
\end{aligned}
$$

For the choice of our metric and form fields we get the following components of Ricci tensor

$$
\begin{aligned}
R_{\mu\nu} &= -\eta_{\mu\nu}\Big[\frac{3A'^2}{B^2} - \frac{AA'B'}{B^3} + \frac{AA'C'}{B^2C} + \frac{2AA'D'}{B^2D} + \frac{2AA'E'}{B^2E} + \frac{AA''}{B^2}\Big] \\
&= -\eta_{\mu\nu}\Big[\frac{(g_sM\alpha')^4}{64}\frac{\ell^2A^2}{C^2D^4E^4} + \frac{g_sM^2\alpha'^2}{32}A^2e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^4} + \frac{g_s^2k'^2}{B^2D^4} + \\
&\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2} + e^{2\Phi}\frac{(1-F)^2}{C^2D^4} + e^{2\Phi}\frac{F^2}{C^2E^4} + 2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big)\Big]. \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
R_{\tau\tau} &= \Big(\frac{4A'B'}{AB} + \frac{B'C'}{BC} + \frac{2B'D'}{BD} + \frac{2B'E'}{BE} - \frac{4A''}{A} - \frac{C''}{C} - \frac{2D''}{D} - \frac{2E''}{E}\Big) \\
&= \frac{1}{2}\Phi'^2 - \frac{(g_sM\alpha')^4}{64}\frac{\ell^2B^2}{C^2D^4E^4} + \frac{g_sM^2\alpha'^2}{8}e^{-\Phi}\Big(\frac{g_s^2f'^2}{E^4} + \frac{g_s^2k'^2}{D^4} + 2e^{2\Phi}\frac{F'^2}{D^2E^2}\Big) - \\
&\frac{g_sM^2\alpha'^2}{32}e^{-\Phi}\Big(\frac{g_s^2f'^2}{E^4} + \frac{g_s^2k'^2}{D^4} + \frac{(k-f)^2}{2}\frac{g_s^2B^2}{C^2D^2E^2} + e^{2\Phi}\frac{(1-F)^2B^2}{C^2D^4} + \\
&e^{2\Phi}\frac{F^2B^2}{C^2E^4} + 2e^{2\Phi}\frac{F'^2}{D^2E^2}\Big) \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
R_{\theta_1\theta_1} &= R_{\theta_2\theta_2} = 1 - \frac{C^2}{4D^2} - \frac{D^2}{16C^2} - \frac{C^2}{4E^2} + \frac{D^4}{16C^2E^2} - \frac{E^2}{16C^2} + \frac{E^4}{16C^2D^2} - \frac{2DA'D'}{AB^2} + \\
&\frac{DB'D'}{2B^3} - \frac{DC'D'}{2B^2C} - \frac{D'^2}{2B^2} - \frac{2EA'E'}{AB^2} + \frac{EB'E'}{2B^3} - \frac{EC'E'}{2B^2C} - \frac{DD'E'}{B^2E} - \\
&\frac{ED'E'}{B^2D} - \frac{E'^2}{2B^2} - \frac{DD''}{2B^2} - \frac{EE''}{2B^2} \\
&= \frac{(g_sM\alpha')^4}{128}\frac{\ell^2(D^2+E^2)}{C^2D^4E^4} + \frac{g_sM^2\alpha'^2}{16}e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^2} + \frac{g_s^2k'^2}{B^2D^2} + \frac{g_s^2(k-f)^2}{4C^2D^2} +
\end{aligned}
$$

$$\frac{g_s^2(k-f)^2}{4C^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^2}+e^{2\Phi}\frac{F^2}{C^2E^2}+e^{2\Phi}\frac{F'^2}{B^2D^2}+e^{2\Phi}\frac{F'^2}{B^2E^2}\Big)-\frac{g_sM^2\alpha'^2}{64}$$
$$e^{-\Phi}(D^2+E^2)\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^4}$$
$$+e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big). \tag{21}$$

$$R_{\phi_i\phi_i}=sin^2\theta_iR_{\theta_i\theta_i}+cos^2\theta_iR_{\psi\psi} \tag{22}$$

$$\begin{aligned}
R_{\psi\psi} &= \Big[\frac{1}{2}+\frac{C^4}{D^2E^2}-\frac{D^2}{4E^2}-\frac{E^2}{4D^2}-\frac{4CA'C'}{AB^2}+\frac{CB'C'}{B^3}-\frac{2CC'D'}{B^2D}-\frac{2CC'E'}{B^2E}-\frac{CC''}{B^2}\Big]\\
&= \frac{(g_sM\alpha')^4}{64}\frac{\ell^2}{D^4E^4}+\frac{g_sM^2\alpha'^2}{16}e^{-\Phi}\Big(\frac{g_s^2(k-f)^2}{D^2E^2}+e^{2\Phi}\frac{2(1-F)^2}{D^4}+e^{2\Phi}\frac{2F^2}{E^4}\Big)-\\
&\quad \frac{g_sM^2\alpha'^2}{32}e^{-\Phi}C^2\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^4}\\
&\quad +e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big).
\end{aligned} \tag{23}$$

$$\begin{aligned}
R_{\psi\phi_i} &= cos\ \theta_i\Big[\frac{1}{2}+\frac{C^4}{D^2E^2}-\frac{D^2}{4E^2}-\frac{E^2}{4D^2}-\frac{4CA'C'}{AB^2}+\frac{CB'C'}{B^3}\\
&\quad -\frac{2CC'D'}{B^2D}-\frac{2CC'E'}{B^2E}-\frac{CC''}{B^2}\Big]\\
&= cos\ \theta_i\Big[\frac{(g_sM\alpha')^4}{64}\frac{\ell^2}{D^4E^4}+\frac{g_sM^2\alpha'^2}{16}e^{-\Phi}\Big(\frac{g_s^2(k-f)^2}{D^2E^2}+e^{2\Phi}\frac{2(1-F)^2}{D^4}+e^{2\Phi}\frac{2F^2}{E^4}\Big)-\\
&\quad \frac{g_sM^2\alpha'^2}{32}e^{-\Phi}C^2\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^4}\\
&\quad +e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big)\Big]=cos\ \theta_iR_{\psi\psi}.
\end{aligned} \tag{24}$$

$$\begin{aligned}
R_{\phi_1\phi_2} &= -sin\theta_1sin\theta_2cos\psi\Big[\frac{C^2}{4D^2}+\frac{D^2}{16C^2}-\frac{C^2}{4E^2}+\frac{D^4}{16C^2E^2}-\frac{E^2}{16C^2}-\frac{E^4}{16C^2D^2}-\\
&\quad \frac{2DA'D'}{AB^2}+\frac{DB'D'}{2B^3}-\frac{DC'D'}{2B^2C}-\frac{D'^2}{2B^2}+\frac{2EA'E'}{AB^2}-\frac{EB'E'}{2B^3}+\frac{EC'E'}{2B^2C}-
\end{aligned}$$

$$
\begin{aligned}
& \frac{DD'E'}{B^2E} + \frac{ED'E'}{B^2D} + \frac{E'^2}{2B^2} - \frac{DD''}{2B^2} + \frac{EE''}{2B^2}\Big] + cos\ \theta_1 cos\theta_2\Big[\frac{1}{2} + \frac{C^4}{D^2E^2} - \\
& \frac{D^2}{4E^2} - \frac{E^2}{4D^2} - \frac{4CA'C'}{AB^2} + \frac{CB'C'}{B^3} - \frac{2CC'D'}{B^2D} - \frac{2CC'E'}{B^2E} - \frac{CC''}{B^2}\Big] \\
= \ & \frac{1}{2}sin\theta_1 sin\theta_2 cos\psi\Big[\frac{(g_sM\alpha')^4}{64}\frac{\ell^2(E^2-D^2)}{C^2D^4E^4} + \frac{g_sM^2\alpha'^2}{8}e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^2} - \frac{g_s^2k'^2}{B^2D^2} + \\
& \frac{g_s^2(k-f)^2}{4C^2D^2} - \frac{g_s^2(k-f)^2}{4C^2E^2} - e^{2\Phi}\frac{(1-F)^2}{C^2D^2} + e^{2\Phi}\frac{F^2}{C^2E^2} + e^{2\Phi}\frac{F'^2}{B^2D^2} - e^{2\Phi}\frac{F'^2}{B^2E^2}\Big) \\
& -\frac{g_sM^2\alpha'^2}{32}e^{-\Phi}(E^2-D^2)\Big(\frac{g_s^2f'^2}{B^2E^4} + \frac{g_s^2k'^2}{B^2D^4} + \frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2} + e^{2\Phi}\frac{(1-F)^2}{C^2D^4} \\
& +e^{2\Phi}\frac{F^2}{C^2E^4} + 2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big)\Big] + cos\theta_1 cos\theta_2\ R_{\psi\psi}
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
R_{\phi_1\theta_2} = \ & sin\theta_1 sin\psi\Big[\frac{C^2}{4D^2} + \frac{D^2}{16C^2} - \frac{C^2}{4E^2} + \frac{D^4}{16C^2E^2} - \frac{E^2}{16C^2} - \frac{E^4}{16C^2D^2} - \frac{2DA'D'}{AB^2} \\
& +\frac{DB'D'}{2B^3} - \frac{DC'D'}{2B^2C} - \frac{D'^2}{2B^2} + \frac{2EA'E'}{AB^2} - \frac{EB'E'}{2B^3} + \frac{EC'E'}{2B^2C} - \\
& \frac{DD'E'}{B^2E} + \frac{ED'E'}{B^2D} + \frac{E'^2}{2B^2} - \frac{DD''}{2B^2} + \frac{EE''}{2B^2}\Big] \\
= \ & -\frac{1}{2}sin\theta_1 sin\psi\Big[\frac{(g_sM\alpha')^4}{64}\frac{\ell^2(E^2-D^2)}{C^2D^4E^4} + \frac{g_sM^2\alpha'^2}{8}e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^2} - \frac{g_s^2k'^2}{B^2D^2} + \\
& \frac{g_s^2(k-f)^2}{4C^2D^2} - \frac{g_s^2(k-f)^2}{4C^2E^2} - e^{2\Phi}\frac{(1-F)^2}{C^2D^2} + e^{2\Phi}\frac{F^2}{C^2E^2} + e^{2\Phi}\frac{F'^2}{B^2D^2} - e^{2\Phi}\frac{F'^2}{B^2E^2}\Big) \\
& -\frac{g_sM^2\alpha'^2}{32}e^{-\Phi}(E^2-D^2)\Big(\frac{g_s^2f'^2}{B^2E^4} + \frac{g_s^2k'^2}{B^2D^4} + \frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2} + e^{2\Phi}\frac{(1-F)^2}{C^2D^4} \\
& +e^{2\Phi}\frac{F^2}{C^2E^4} + 2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big)\Big]
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
R_{\theta_1\theta_2} = \ & cos\psi\Big[\frac{C^2}{4D^2} + \frac{D^2}{16C^2} - \frac{C^2}{4E^2} + \frac{D^4}{16C^2E^2} - \frac{E^2}{16C^2} - \frac{E^4}{16C^2D^2} - \frac{2DA'D'}{AB^2} \\
& +\frac{DB'D'}{2B^3} - \frac{DC'D'}{2B^2C} - \frac{D'^2}{2B^2} + \frac{2EA'E'}{AB^2} - \frac{EB'E'}{2B^3} + \frac{EC'E'}{2B^2C} - \\
& \frac{DD'E'}{B^2E} + \frac{ED'E'}{B^2D} + \frac{E'^2}{2B^2} - \frac{DD''}{2B^2} + \frac{EE''}{2B^2}\Big]
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{2}cos\psi\Big[\frac{(g_sM\alpha')^4}{64}\frac{\ell^2(E^2-D^2)}{C^2D^4E^4}+\frac{g_sM^2\alpha'^2}{8}e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^2}-\frac{g_s^2k'^2}{B^2D^2}+\\
&\frac{g_s^2(k-f)^2}{4C^2D^2}-\frac{g_s^2(k-f)^2}{4C^2E^2}-e^{2\Phi}\frac{(1-F)^2}{C^2D^2}+e^{2\Phi}\frac{F^2}{C^2E^2}+e^{2\Phi}\frac{F'^2}{B^2D^2}-e^{2\Phi}\frac{F'^2}{B^2E^2}\Big)\\
&-\frac{g_sM^2\alpha'^2}{32}e^{-\Phi}(E^2-D^2)\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^4}\\
&+e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big)\Big] \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
R_{\phi_2\theta_1} &= sin\theta_2 sin\psi\Big[\frac{C^2}{4D^2}+\frac{D^2}{16C^2}-\frac{C^2}{4E^2}+\frac{D^4}{16C^2E^2}-\frac{E^2}{16C^2}-\frac{E^4}{16C^2D^2}-\frac{2DA'D'}{AB^2}\\
&+\frac{DB'D'}{2B^3}-\frac{DC'D'}{2B^2C}-\frac{D'^2}{2B^2}+\frac{2EA'E'}{AB^2}-\frac{EB'E'}{2B^3}+\frac{EC'E'}{2B^2C}-\\
&\frac{DD'E'}{B^2E}+\frac{ED'E'}{B^2D}+\frac{E'^2}{2B^2}-\frac{DD''}{2B^2}+\frac{EE''}{2B^2}\Big]\\
&= -\frac{1}{2}sin\theta_2 sin\psi\Big[\frac{(g_sM\alpha')^4}{64}\frac{\ell^2(E^2-D^2)}{C^2D^4E^4}+\frac{g_sM^2\alpha'^2}{8}e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^2}-\frac{g_s^2k'^2}{B^2D^2}+\\
&\frac{g_s^2(k-f)^2}{4C^2D^2}-\frac{g_s^2(k-f)^2}{4C^2E^2}-e^{2\Phi}\frac{(1-F)^2}{C^2D^2}+e^{2\Phi}\frac{F^2}{C^2E^2}+e^{2\Phi}\frac{F'^2}{B^2D^2}-e^{2\Phi}\frac{F'^2}{B^2E^2}\Big)\\
&-\frac{g_sM^2\alpha'^2}{32}e^{-\Phi}(E^2-D^2)\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^4}\\
&+e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big)\Big] \qquad (28)
\end{aligned}
$$

The Ricci scalar is

$$
\begin{aligned}
R &= \frac{1}{2C^2}+\frac{2}{D^2}+\frac{2}{E^2}-\frac{C^2}{D^2E^2}-\frac{D^2}{4C^2E^2}-\frac{E^2}{4C^2D^2}-\frac{12A'^2}{A^2B^2}+\frac{8A'B'}{AB^3}-\\
&\frac{8A'C'}{AB^2C}+\frac{2B'C'}{B^3C}-\frac{16A'D'}{AB^2D}+\frac{4B'D'}{B^3D}-\frac{4C'D'}{B^2CD}-\frac{2D'^2}{B^2D^2}-\frac{16A'E'}{AB^2E}+\\
&\frac{4B'E'}{B^3E}-\frac{4C'E'}{B^2CE}-\frac{8D'E'}{B^2DE}-\frac{2E'^2}{B^2E^2}-\frac{8A''}{AB^2}-\frac{2C''}{B^2C}-\frac{4D''}{B^2D}-\frac{4E''}{B^2E}\\
&= \frac{1}{2}\frac{\Phi'^2}{B^2}+\frac{g_sM^2\alpha'^2}{16}e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{g_s^2(k-f)^2}{2C^2D^2E^2}+\\
&e^{2\Phi}\frac{(1-F)^2}{C^2D^4}+e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big) \qquad (29)
\end{aligned}
$$

It just follows from the computation of the Ricci components that all the components are not independent, which tells us to make a choice. So we shall take the Ricci components that give independent equations are

$$R_{xx}, R_{\tau\tau}, R_{\theta_1\theta_1}, R_{\psi\psi}, R_{\theta_1\theta_2}. \tag{30}$$

It is interesting to note that some of the Ricci components that are computed using the metric eq(6) are symmetric and some are anti-symmetric under the interchange of $D \longleftrightarrow E$, that is the size of the two S^2s.

Upon going through the ansatz to the solution, we found that there are 9 unknowns: one from F_3 flux, two from H_3 flux, one from dilaton and five from the metric. Simultaneously there are 9 equations: one from F_3 equation, two from H_3, one from dilaton and five from Ricci tensor equations. So there are as many equations as unknowns.

Let us summarize all the 9 equations:

$$[\mathbf{1}]\ \frac{1}{A^4BCD^2E^2}\partial_\tau[\frac{A^4CD^2E^2\Phi'}{B}] = -\frac{g_sM^2\alpha'^2}{8}e^{-\Phi}[\frac{g_s^2f'^2}{B^2E^4} + \frac{g_sk'^2}{B^2D^4} +$$
$$\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2} - e^{2\Phi}\frac{(1-F)^2}{C^2D^4} - e^{2\Phi}\frac{F^2}{C^2E^4} - 2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}],$$

$$[\mathbf{2}]\ \frac{1-F}{2}e^{\Phi}\frac{A^4BE^2}{CD^2} - \frac{F}{2}e^{\Phi}\frac{A^4BD^2}{CE^2} + \partial_\tau(F'e^{\Phi}\frac{A^4C}{B}) = \frac{g_s^3M^2\alpha'^2}{4}\frac{A^4B}{CD^2E^2}\frac{\ell(k-f)}{2}$$

$$[\mathbf{3}]\ \frac{g_sM^2\alpha'^2}{2}\frac{A^4B}{CD^2E^2}\ell(1-F) = 2\partial_\tau(f'e^{-\Phi}\frac{A^4CD^2}{BE^2}) + (k-f)e^{-\Phi}\frac{A^4B}{C}$$

$$[\mathbf{4}]\ \frac{g_sM^2\alpha'^2}{2}\frac{A^4B}{CD^2E^2}F\ell = 2\partial_\tau(k'e^{-\Phi}\frac{A^4CE^2}{BD^2}) - (k-f)e^{-\Phi}\frac{A^4B}{C},$$

$$[\mathbf{5}]\ \frac{3A'^2}{B^2} - \frac{AA'B'}{B^3} + \frac{AA'C'}{B^2C} + \frac{2AA'D'}{B^2D} + \frac{2AA'E'}{B^2E} + \frac{AA''}{B^2} =$$
$$\frac{(g_sM\alpha')^4}{64}\frac{\ell^2A^2}{C^2D^4E^4} + \frac{g_sM^2\alpha'^2}{32}A^2e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^4} + \frac{g_s^2k'^2}{B^2D^4} +$$
$$\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2} + e^{2\Phi}\frac{(1-F)^2}{C^2D^4} + e^{2\Phi}\frac{F^2}{C^2E^4} + 2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big),$$

$$[\mathbf{6}]\ \frac{4A'B'}{AB} + \frac{B'C'}{BC} + \frac{2B'D'}{BD} + \frac{2B'E'}{BE} - \frac{4A''}{A} - \frac{C''}{C} - \frac{2D''}{D} - \frac{2E''}{E} =$$
$$\frac{1}{2}\Phi'^2 - \frac{(g_sM\alpha')^4}{64}\frac{\ell^2B^2}{C^2D^4E^4} + \frac{g_sM^2\alpha'^2}{8}e^{-\Phi}\Big(\frac{g_s^2f'^2}{E^4} + \frac{g_s^2k'^2}{D^4} + 2e^{2\Phi}\frac{F'^2}{D^2E^2}\Big) -$$

$$\frac{g_sM^2\alpha'^2}{32}e^{-\Phi}\Big(\frac{g_s^2f'^2}{E^4}+\frac{g_s^2k'^2}{D^4}+\frac{(k-f)^2}{2}\frac{g_s^2B^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2B^2}{C^2D^4}+$$
$$e^{2\Phi}\frac{F^2B^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{D^2E^2}\Big),$$

$$[\mathbf{7}]\ 1-\frac{C^2}{4D^2}-\frac{D^2}{16C^2}-\frac{C^2}{4E^2}+\frac{D^4}{16C^2E^2}-\frac{E^2}{16C^2}+\frac{E^4}{16C^2D^2}-\frac{2DA'D'}{AB^2}+$$
$$\frac{DB'D'}{2B^3}-\frac{DC'D'}{2B^2C}-\frac{D'^2}{2B^2}-\frac{2EA'E'}{AB^2}+\frac{EB'E'}{2B^3}-\frac{EC'E'}{2B^2C}-\frac{DD'E'}{B^2E}-$$
$$\frac{ED'E'}{B^2D}-\frac{E'^2}{2B^2}-\frac{DD''}{2B^2}-\frac{EE''}{2B^2}$$
$$=\quad\frac{(g_sM\alpha')^4}{128}\frac{\ell^2(D^2+E^2)}{C^2D^4E^4}+\frac{g_sM^2\alpha'^2}{16}e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^2}+\frac{g_s^2k'^2}{B^2D^2}+\frac{g_s^2(k-f)^2}{4C^2D^2}+$$
$$\frac{g_s^2(k-f)^2}{4C^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^2}+e^{2\Phi}\frac{F^2}{C^2E^2}+e^{2\Phi}\frac{F'^2}{B^2D^2}+e^{2\Phi}\frac{F'^2}{B^2E^2}\Big)-\frac{g_sM^2\alpha'^2}{64}$$
$$e^{-\Phi}(D^2+E^2)\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^4}$$
$$+e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big),$$

$$[\mathbf{8}]\ \frac{1}{2}+\frac{C^4}{D^2E^2}-\frac{D^2}{4E^2}-\frac{E^2}{4D^2}-\frac{4CA'C'}{AB^2}+\frac{CB'C'}{B^3}-\frac{2CC'D'}{B^2D}-\frac{2CC'E'}{B^2E}-\frac{CC''}{B^2}$$
$$=\quad\frac{(g_sM\alpha')^4}{64}\frac{\ell^2}{D^4E^4}+\frac{g_sM^2\alpha'^2}{16}e^{-\Phi}\Big(\frac{g_s^2(k-f)^2}{D^2E^2}+e^{2\Phi}\frac{2(1-F)^2}{D^4}+e^{2\Phi}\frac{2F^2}{E^4}\Big)-$$
$$\frac{g_sM^2\alpha'^2}{32}e^{-\Phi}C^2\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^4}$$
$$+e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big),$$

$$[\mathbf{9}]\ \frac{C^2}{4D^2}+\frac{D^2}{16C^2}-\frac{C^2}{4E^2}+\frac{D^4}{16C^2E^2}-\frac{E^2}{16C^2}-\frac{E^4}{16C^2D^2}-\frac{2DA'D'}{AB^2}$$
$$+\frac{DB'D'}{2B^3}-\frac{DC'D'}{2B^2C}-\frac{D'^2}{2B^2}+\frac{2EA'E'}{AB^2}-\frac{EB'E'}{2B^3}+\frac{EC'E'}{2B^2C}-$$
$$\frac{DD'E'}{B^2E}+\frac{ED'E'}{B^2D}+\frac{E'^2}{2B^2}-\frac{DD''}{2B^2}+\frac{EE''}{2B^2}$$
$$=\quad-\frac{1}{2}\Big[\frac{(g_sM\alpha')^4}{64}\frac{\ell^2(E^2-D^2)}{C^2D^4E^4}+\frac{g_sM^2\alpha'^2}{8}e^{-\Phi}\Big(\frac{g_s^2f'^2}{B^2E^2}-\frac{g_s^2k'^2}{B^2D^2}+$$

$$
\frac{g_s^2(k-f)^2}{4C^2D^2}-\frac{g_s^2(k-f)^2}{4C^2E^2}-e^{2\Phi}\frac{(1-F)^2}{C^2D^2}+e^{2\Phi}\frac{F^2}{C^2E^2}+e^{2\Phi}\frac{F'^2}{B^2D^2}-e^{2\Phi}\frac{F'^2}{B^2E^2}\Big)
-\frac{g_sM^2\alpha'^2}{32}e^{-\Phi}(E^2-D^2)\Big(\frac{g_s^2f'^2}{B^2E^4}+\frac{g_s^2k'^2}{B^2D^4}+\frac{(k-f)^2}{2}\frac{g_s^2}{C^2D^2E^2}+e^{2\Phi}\frac{(1-F)^2}{C^2D^4}
+e^{2\Phi}\frac{F^2}{C^2E^4}+2e^{2\Phi}\frac{F'^2}{B^2D^2E^2}\Big)\Big] \tag{31}
$$

3 Solutions

There exists three solutions to these equations and are known as KT, KS and ABY/DKM. These solutions when expressed in our parametrization, reads as

KT:

$$
\begin{aligned}
&A=B^{-1}=h^{-\frac{1}{4}},\quad \tau=r,\quad C^2=h^{\frac{1}{2}}\frac{r^2}{9},\quad D^2=E^2=h^{\frac{1}{2}}\frac{r^2}{6},\\
&\Phi=Log\ g_s, F=\frac{1}{2},\quad f=k=\frac{3}{2}Log\ \frac{r}{r_0},\quad \ell=\frac{3}{2}Log\frac{r}{r_0}\\
&h(r)=\frac{27\pi g_s\alpha'^2}{4r^4}[\frac{3g_sM^2}{2\pi}Log\ \frac{r}{r_0}+\frac{3g_sM^2}{8\pi}]
\end{aligned} \tag{32}
$$

KS:

$$
\begin{aligned}
&A^2=h^{-\frac{1}{2}},\quad B^2=C^2=\frac{h^{\frac{1}{2}}\varepsilon^{\frac{4}{3}}}{6K^2},\quad D^2=\frac{h^{\frac{1}{2}}\varepsilon^{\frac{4}{3}}}{2}Kcosh^2\frac{\tau}{2},\quad E^2=\frac{h^{\frac{1}{2}}\varepsilon^{\frac{4}{3}}}{2}Ksinh^2\frac{\tau}{2},\\
&K=\frac{(sinh2\tau-2\tau)^{\frac{1}{3}}}{2^{\frac{1}{3}}sinh\tau},\Phi=Log\ g_s,F=\frac{sinh\tau-\tau}{2sinh\tau},\quad f=\frac{\tau coth\tau-1}{2sinh\tau}(cosh\tau-1),\\
&k=\frac{\tau coth\tau-1}{2sinh\tau}(cosh\tau+1),\quad \ell=\frac{\tau coth\tau-1}{4sinh^2\tau}(sinh2\tau-2\tau),\\
&h(\tau)=(g_sM\alpha')^22^{\frac{2}{3}}\varepsilon^{-\frac{8}{3}}I(\tau),\quad I(\tau)=\int_\tau^\infty dx\frac{xcoth\ x-1}{sinh^2x}(sinh2x-2x)^{\frac{1}{3}},\\
&I(\tau)\approx 0.71805+0.18344\ \tau^2-0.0306\ \tau^4+\cdots
\end{aligned} \tag{33}
$$

ABY/DKM:

The parametrization that we used is related to DKM as

$$A^2 = B^{-2} = r^2 e^{2a}, \quad C^2 = \frac{e^{2b-2a}}{9}, \quad D^2 = E^2 = \frac{e^{2c-2a}}{6}, \quad \tau = r$$
$$F = \frac{1}{2}, \quad 3(g_s M\alpha')f = 3(g_s M\alpha')k = k_{DKM} \tag{34}$$

If we assume that the unknown functions that appear in B_2 and metric satisfies the following conditions i.e. $f = k$ and $D^2 = E^2$ then there is a simple solution to eq.(17) that is $F = \frac{1}{2}$, which is consistent with the flux quantization condition for F_3 and is in fact the solution for KT and ABY/DKM solutions. For this special case we left with 6 unknowns and as many equations.

Let us try to find the linearized solution to the rest of the equations of motion written above, for this special case. Upon assuming that the solution depends on two parameters $\mathcal{S}$ and ϕ, explicitly it means that the solution reads as

$$h(r) = \frac{27\pi g_s\alpha'^2}{4r^4}[\frac{3g_sM^2}{2\pi}Log\ \frac{r}{r_0} + \frac{3g_sM^2}{8\pi}] +$$
$$\frac{(g_SM\alpha')^2}{r^8}[(h_1 + h_2 Log\ r)\mathcal{S} - h_3\phi]$$
$$A(r) = h^{-\frac{1}{4}} = B^{-1}, \quad C(r) = \frac{r}{3}h(r)^{\frac{1}{4}}[1 + \frac{1}{r^4}\Big((c_1 + c_2 Log\ r)\mathcal{S} - c_3\phi\Big)]$$
$$D(r) = \frac{r}{\sqrt{6}}h(r)^{\frac{1}{4}}[1 + \frac{1}{r^4}\Big((d_1 + d_2 Log\ r)\mathcal{S} - d_3\phi\Big)] = E(r)$$
$$\Phi(r) = Log\ g_s + \frac{1}{r^4}\Big((p_1 + p_2 Log\ r)\mathcal{S} - p_3\phi\Big)$$
$$f(r) = k(r) = \ell(r) = \frac{3}{2}Log\ r + \frac{1}{r^4}\Big((f_1 + f_2 Log\ r)\mathcal{S} - f_3\phi\Big), \tag{35}$$

where $c_1,\ c_2,\ c_3,\ d_1,\ d_2,\ d_3,\ f_1,\ f_2,\ f_3,\ h_1,\ h_2,\ h_3,\ p_1,\ p_2, p_3$ are all constants. Now solving these 6 equations we get the answer which depends on $h_1,\ h_2$ and h_3 as

$$c_1 = \frac{8}{81}h_2, \quad c_2 = 0, \quad c_3 = 0, \quad d_1 = 0, \quad d_2 = 0, \quad d_3 = 0, ,$$
$$p_1 = -\frac{64}{81}h_1 + \frac{52}{81}h_2, \quad p_2 = -\frac{16}{27}h_2, \quad p_3 = -\frac{64}{81}h_3,$$
$$f_1 = \frac{8}{27}h_1 - \frac{h_2}{27}, \quad f_2 = \frac{4}{9}h_2, \quad f_3 = \frac{8}{27}h_3, \tag{36}$$

it essentially means that this linearized perturbation generates solution which is characterized completely by the warp factor.

For a choice like: $h_1 = \frac{1053}{256}$, $h_2 = \frac{81}{16}$, and $h_3 = \frac{81}{64}$, we get back the DKM solution [16]. Now, the question arises why such a choice is special ?

To answer this question we may need to look at the supersymmetry preserved by the solution. This we can say by looking at the type of the complex combination of three form flux that comes from RR, F_3 and NS-NS sector, H_3.

The general form of the complex 3-form

$$\begin{aligned} G_3 &= F_3 - ie^{-\Phi}H_3 \\ &= \frac{M\alpha'}{2}[(1-F)g_5\wedge g_3\wedge g_4 + Fg_5\wedge g_1\wedge g_2 - ie^{-\Phi}g_s\frac{(k-f)}{2}g_5\wedge g_1\wedge g_3 - \\ & ie^{-\Phi}g_s\frac{(k-f)}{2}g_5\wedge g_2\wedge g_4 + F'd\tau\wedge g_1\wedge g_3 + F'd\tau\wedge g_2\wedge g_4 - \\ & ie^{-\Phi}g_sf'd\tau\wedge g_1\wedge g_2 - ie^{-\Phi}g_sk'd\tau\wedge g_3\wedge g_4] \end{aligned} \tag{37}$$

For KT case it reduces to

$$G_3 = \frac{M\alpha'}{4}(g_5 - 2ie^{-\Phi}g_srf'\frac{dr}{r})\wedge(g_1\wedge g_2 + g_3\wedge g_4) \tag{38}$$

Using the solution eq(36), the metric for KT case reads

$$ds^2 = h^{-\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + r^2h^{\frac{1}{2}}[\frac{dr^2}{r^2} + (1+2\frac{c_1\mathcal{S}}{r^4})\frac{g_5^2}{9} + \frac{g_1^2+g_2^2+g_3^2+g_4^2}{6}] \tag{39}$$

From this metric it just follows that we can introduce complex coordinate as

$$\frac{dr}{r} + i(1+\frac{c_1\mathcal{S}}{r^4})\frac{g_5}{3} = \frac{2}{3r^3}\overline{z}_idz_i, \quad \frac{dr}{r} - i(1+\frac{c_1\mathcal{S}}{r^4})\frac{g_5}{3} = \frac{2}{3r^3}z_i\overline{dz}_i, \tag{40}$$

and from the paper of [17]

$$g_1\wedge g_2 + g_3\wedge g_4 = \frac{2i}{r^6}\epsilon_{ijkl}z_i\overline{z}_jdz_k\wedge\overline{dz}_l. \tag{41}$$

combining all that we get the expression

$$\begin{aligned} G_3 &= \frac{M\alpha'}{2r^9}\Big[\Big((1-\frac{c_1\mathcal{S}}{r^4}) + \frac{2}{3}e^{-\Phi}g_srf'\Big)\overline{z}_mdz_m\wedge\epsilon_{ijkl}z_i\overline{z}_jdz_k\wedge\overline{dz}_l - \\ & \Big((1-\frac{c_1\mathcal{S}}{r^4}) - \frac{2}{3}e^{-\Phi}g_srf'\Big)z_m\overline{dz}_m\wedge\epsilon_{ijkl}z_i\overline{z}_jdz_k\wedge\overline{dz}_l\Big] \\ &= \frac{1}{2i}(G_+ + G_-) \end{aligned} \tag{42}$$

It just follows trivially that

$$
\begin{aligned}
G_+ &= \frac{iM\alpha'}{r^9}\Big((1-\frac{c_1\mathcal{S}}{r^4})+\frac{2}{3}e^{-\Phi}g_s r f'\Big)\\
&= \frac{iM\alpha'}{r^9}\Big[2-\frac{\mathcal{S}h_2}{r^4}\Big(\frac{28}{81}+\frac{16}{27}Log\ r\Big)\Big]\overline{z}_m dz_m\wedge\epsilon_{ijkl}z_i\overline{z}_j dz_k\wedge\overline{dz}_l\\
G_- &= -\frac{iM\alpha'}{r^9}\Big((1-\frac{c_1\mathcal{S}}{r^4})-\frac{2}{3}e^{-\Phi}g_s r f'\Big)\\
&= -\frac{iM\alpha'}{r^9}\Big[\frac{\mathcal{S}h_2}{r^4}\Big(\frac{12}{81}+\frac{16}{27}Log\ r\Big)\Big]z_m\overline{dz}_m\wedge\epsilon_{ijkl}z_i\overline{z}_j dz_k\wedge\overline{dz}_l,
\end{aligned}
\tag{43}
$$

where G_+ is a (2, 1) form and G_- is (1, 2) form and its computed using the back reacted metric. Now it is easy to draw the conclusion that for $h_2 = 0$ we can have a supersymmetry preserving solution. Which is not surprising, as the metric of the conifold eq(39) has changed to terms proportional to $c_1 \sim h_2$.

Probably it makes sense to say that we have a two dimensional real space that is described by (h_1, h_3) plus a point at the origin $h_2 = 0$, preserves supersymmetry. As soon as we go away from the origin along the h_2 axis the system is not any more supersymmetric. Hence the interpretation that we have a supersymmetry preserving plane is correct.

After taking the back reaction of the fluctuation we see that the metric of changed singular metric can be made to be same as the metric of the singular conifold metric upon setting the non-supersymmetric fluctuation to zero. This point could be useful to find the solution of the linearized fluctuation of the deformed conifold [19].

4 conclusion

We have re-visited the linearized perturbation [16] to the gravity solution of the intersecting D3 branes and D5 branes wrapped on a 2 sphere [5], linear in the parameters $\mathcal{S}$ and ϕ and found that there arises infinitely many choices to h_1, h_2 and h_3. For any choice except the choice $(h_1,\ h_3)$ plane sitting at $h_2 = 0$, break supersymmetry dynamically.

For a supersymmetry preserving solution the vacuum energy should vanish, which means from the dual field theory point of view the energy should

vanish and by Lorentz invariance the energy-momentum tensor should go as

$$< T_{\mu\nu} > \quad \sim \quad \eta_{\mu\nu}\ h_2\mathcal{S}. \tag{44}$$

Its exact structure need to be computed following [18], but for the specific choice to h_1, h_2 and h_3 i.e. the DKM solution [16], the authors have given the relation between the parameter $\mathcal{S}$ and the energy momentum tensor, $T_{\mu\nu}$.

Let us recall from the gauge gravity duality for the cascading theory, that is the two gauge couplings are related to the bulk fields

$$\begin{aligned}
&\frac{4\pi^2}{g_1^2} + \frac{4\pi^2}{g_2^2} = \frac{\pi}{g_s} e^{-\Phi} \\
&\Big(\frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2}\Big) g_s e^{\Phi} = \frac{1}{2\pi\alpha'}\Big(\oint_{S^2} B_2\Big) - \pi\ (mod\ 2\pi)
\end{aligned} \tag{45}$$

Now using the solution to the bulk field equations of motion into this duality

$$\begin{aligned}
&\frac{4\pi^2}{g_1^2} + \frac{4\pi^2}{g_2^2} = \frac{\pi}{g_s^2}\Big[1 - \frac{1}{r^4}\Big((-\frac{64}{81}h_1 + \frac{52}{81}h_2)\mathcal{S} - \frac{16}{27}h_2\mathcal{S} Log\ r + \frac{64}{81}h_3\phi\Big)\Big] \\
&\frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2} = \frac{3M}{g_s} Log\ r + \\
&\frac{\mathcal{S}}{r^4}(-\frac{64}{81}h_1 + \frac{52}{81}h_2 - \frac{16}{27}h_2 Log\ r)(\pi - \frac{3M}{g_s} Log\ r) + \\
&\frac{\mathcal{S}}{r^4}\frac{2M}{g_s}\Big((\frac{8}{27}h_1 - \frac{h_2}{27}) + \frac{4}{9}h_2 Log\ r\Big) - \\
&\frac{\phi}{r^4}\Big(- \frac{64}{81}h_3(\pi - \frac{3M}{g_s} Log\ r) + \frac{2M}{g_s}\frac{8}{27}h_3\Big) - \pi\ (mod\ 2\pi)
\end{aligned} \tag{46}$$

The non constancy nature of the dilaton makes that the β function for $\frac{8\pi^2}{g_1^2} + \frac{8\pi^2}{g_2^2}$ do not vanishes any more and $\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_2^2}$ has the leading term that goes logarithmically and the subleading term goes as inverse power law.

Its important to understand the field theory dual of this solution and the connection of it with [20]-[37], if any.

Acknowledgement
I would like to thank Shamit Kachru for useful correspondences, Dipankar Das, Srijesh and the computer center, IOPB for computer related help. I am greatly helped by Hara Lenka and A. Rath. It gives me immense pleasure to thank my family for their love, affection and support. Also I have enjoyed the company of my fellow villagers, Barchana.

References

[1] J. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. 2, 231 (1998), Int. J. Theor. Phys. 38, 1113, (1999), hep-th/9711200.

[2] S. Gubser, I. Klebanov and A. Polyakov "Gauge theory correlators from non-critical string theory," Phys. Lett. B 428, 105 (1998), hep-th/9802109

[3] E. Witten "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. 2, 253 (1998), hep-th/9802150.

[4] I. Klebanov and N. Nekrasov "Gravity duals of fractional branes and logarithmic RG flow," Nucl. Phys. B 574, 263 (2000), hep-th/9911096.

[5] I. Klebanov and A. Tseytlin "Gravity duals of supersymmetric SU(N)x SU(N+M) gauge theories," Nucl. Phys. B 578, 123 (2000), hep-th/0002159.

[6] R. Minasian and D. Tsimpis "On the geometry of non-trivially embedded branes,"hep-th/9911042.

[7] I. Klebanov and M. Strassler "Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularity," JHEP 0008, 052 (2000), hep-th/0007191.

[8] P. Candelas and X. de la Ossa "Comments on conifolds," Nucl. Phys. B 342, 246 (1990).

[9] N. Seiberg "Electric-magnetic duality in supersymmetric non-Abelian gauge theories," Nucl. Phys. B 435, 129 (1995), hep-th/9411149.

[10] M. Strassler "The duality cascade," hep-th/0505153.

[11] S. Giddings, S. Kachru and J. Polchinski "Hierarchies from fluxes in string compactifications," Phys. Rev. D 66, 106006 (2002), hep-th/0105097.

[12] I. Klebanov, P. Ouyang and E. Witten "A gravity dual of the chiral anomaly," Phys. Rev. D 65, 105007 (2002), hep-th/0202056.

[13] A. Uranga "Brane configurations for branes at conifolds," JHEP 01, 022 (1999)

[14] K. Dasgupta and S. Mukhi "Brane constructions, conifolds and M-theory," Nucl. Phys. B 551, 204 (1999), hep-th/9811139.

[15] G. Papadopoulos and A. Tseytlin "Complex geometry of conifolds and 5-brane wrapped on 2-sphere," Class. Quant. Grav. 18 1333 (2001), hep-th/0012034.

[16] O. DeWolfe, S. Kachru and M. Mulligan "A gravity dual of metastable dynamical supersymmetry breaking," arXiv:0801.1520

[17] C. Herzog, I. Klebanov and P. Ouyang "Remarks on the warped deformed conifold," hep-th/0108101.

[18] O. Aharony, A. Buchel and A. Yarom "Holographic renormalization of cascading gauge theories," Phys. Rev. D 72, 066003 (2005), hep-th/0506002.

[19] *Work in progess.*

[20] K. Intriligator, N. Seiberg and D. Shih "Dynamical SUSY breaking in meta-stable vacua," JHEP 0604, 021 (2006), hep-th/0602239.

[21] M. Douglas, J. Shelton and G. Torroba "Warping and supersymmetry breaking," arXiv:0704.4001.

[22] K. Intriligator and S. Thomas "Dual descriptions of supersymmetry breaking," hep-th/9608046.

[23] Z. chacko, M. Luty and E. Ponton "Calculable dynamical supersymmetry breaking on the deformed moduli spaces," JHEP 9812, 016 (1998), hep-th/9810253.

[24] E. Poppitz and S. Trivedi "Dynamical supersymmetry breaking," Ann. Rev. Nucl. Part. Sci. 48, 307 (1998), hep-th/9803107.

[25] K. Izawa and T. Yanagida "Dynamical supersymmetry breaking in vector-like gauge theories," Prog. Theor. Phys. 05 829 (1996), hrp-th/9602180.

[26] S. Kuperstein and J. Sonnenschein "Analytic nonsupersymmetric backgrond dual to a confining gauge theory and the corresponding plane wave theory of hadrons," JHEP 0402, 015 (2004), hep-th/0309011.

[27] R. Argurio, M. Bertolini, S. Franco and S. Kachru "Gauge/gravity duality and metastable dynamical supersymmetry breaking," JHEP 0701, 083 (2007), hep-th/0610212

[28] R. Argurio, M. Bertolini, S. Franco and S. Kachru "Metastable vacua and D-branes at the conifold," JHEP 0706, 017 (2007), hep-th/0703236.

[29] M. Aganagic, C. Beem, J. Seo and C. Vafa "Geometrically induced metastability and holography," Nucl. Phys. B 789, 382 (2008), hep-th/0610249.

[30] J. Heckman, J. Seo and C. Vafa "Phase structure of brane/anti-brane system at large N," JHEP 0707, 073 (2007), hep-th/0702077.

[31] J. Marsano, K. Papadodimas and M. shigemori "Nonsupersymmetric brane/antibrane configurations in type IIA and M theory," arXiv:0705.0983.

[32] O. Aharony, S. Kachru and E. Silverstein "Simple stringy dynamical susy breaking," arXiv:0708.0493.

[33] M. Aganagic, C. Beem and B. Freivogel "Geometric metastability, quivers and holography," arXiv:0708.0596

[34] M. Aganagic, C. Beem and S. Kachru "Geometric transitions and dynamical susy breaking," arXiv:0709.4277

[35] M. Buican, D. Malyshev and H. Verlinde "On the geometry of metastable supersymmetry breaking," arXiv:0710.5519.

[36] G. Shiu, G. Torroba, B. Underwood and M. Douglas "Dynamics of warped flux compactification,"arXiv:0803.3068.

[37] S. Franco, D. Rodriguez-Gomez and H. Verlinde "N-ification of forces: A holographic perspective on D-brane model building,"arXiv:0804.1125.